Lectures on Theoretical Cosmology

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UC San Diego

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Prediction of the CMB

What is the origin of chemical elements and how can one explain their relative abundances?

Extrapolating the expansion rate backwards to energy densities necessary for element formation, Gamow in 1946 writes:

> Returning to our problem of the formation of elements, we see that *the conditions necessary for rapid nuclear reactions were existing only for a very short time*, so that it may be quite dangerous to speak about an equilibrium-state which must have been established during this period.

casting doubt on the previously held idea that the chemical elements formed in an equilibrium process.

Prediction of the CMB

Based on this observation, Alpher, Bethe, Gamow in 1948 propose that elements formed by neutron capture

$$\frac{dn_i}{dt} = f(t)(\sigma_{i-1}n_{i-1} - \sigma_i n_i) \quad i = 1, 2, \cdots 238,$$

With cross sections, and assuming all elements are created through this process, one can fit the observed abundances to determine

$$\int_{t_0}^{t_1} n_n dt$$

or equivalently $\int_{t_0}^{t_1} \rho_n dt$ using $\rho_n = m n_n$

Prediction of the CMB

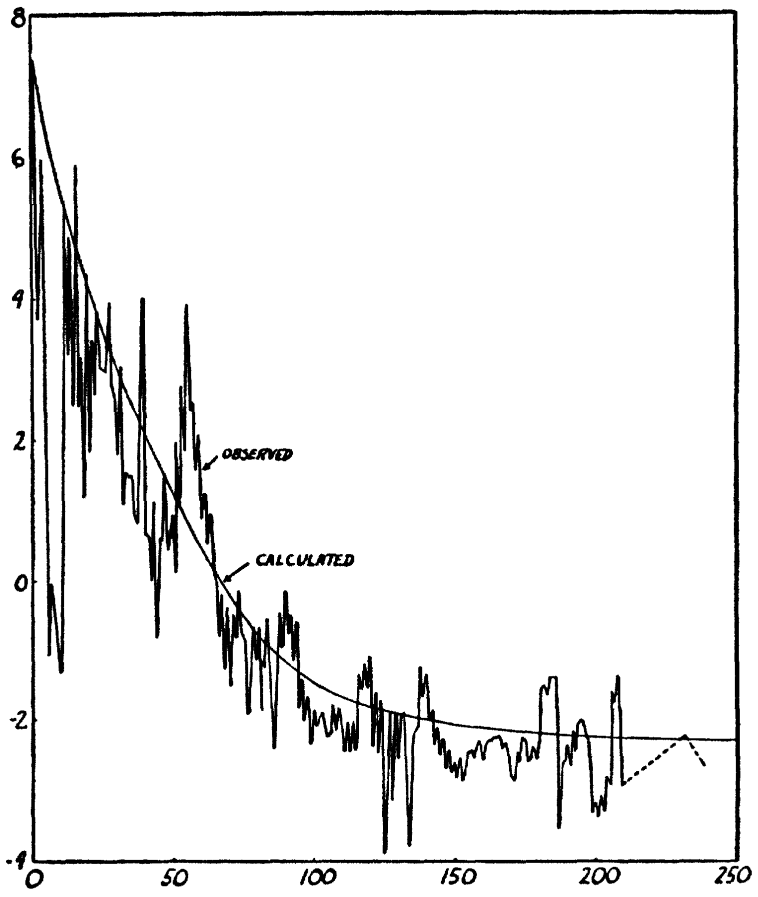

FIG. 1.

Alpher, Bethe, Gamow give a value that is wrong (by 10 orders of magnitude)

Prediction of the CMB

Alpher corrected the mistake in 1948, and finds

$$\int_{t_0}^{t_1} n_n dt = 0.81 \times 10^{18} \frac{s}{cm^3}$$

using this procedure.

If the universe were only filled with nucleons at this time, one would have

$$n_n \approx \frac{\rho e^{-t/\tau_n}}{m_n} \quad \text{with} \quad \rho = \frac{3H^2}{8\pi G} = \frac{1}{6\pi G t^2}$$

$$\text{and} \quad \int_{t_0}^{t_1} n_n dt = \int_{t_0}^{t_1} \frac{e^{-t/\tau_n}}{6\pi G m_n t^2}$$

Prediction of the CMB

Assuming the process takes a time comparable to the neutron lifetime, the observed abundances imply a start time

$$t_0 \approx 10^4 s \gg \tau_n$$

The universe would consist of only hydrogen!

As Alpher points out, a hot big bang in which the universe is filled with black body radiation in addition to matter at the time of element formation provides a way out.

Prediction of the CMB

A flaw with these estimates is that the gap at A=5,8 implies that the heavy elements cannot be formed by neutron capture in the early universe.

Gamow 1948 provides an alternative estimate that is on the right track.

Before heavy elements can form, deuterium must form.

$$n_{n,p}\sigma_n v \sim H$$

Prediction of the CMB

with the known capture cross section for fast neutrons on hydrogen

$$\sigma_n \sim 4 \times 10^{-29} cm^2$$

and velocity $$v \sim 10^9 cm/s$$

this implies

$$n_n t \sim \frac{1}{\sigma_n v} \sim 10^{20} \frac{s}{cm^3}$$

In a matter dominated universe this again implies a start time

$$t_0 \approx 10^4 s \gg \tau_n$$

and a universe filled only with hydrogen.

Prediction of the CMB

Based on this both Alpher and Gamow consider a hot big bang with a universe dominated by black radiation at early times.

An estimate of the temperature of this radiation today is also given

In fact, we find that the value of $\rho_{r''}$ consistent with Eq. (4) is

$$\rho_{r''} \cong 10^{-32}\ \mathrm{g/cm^3}, \qquad (12d)$$

which corresponds to a temperature now of the order of 5°K.

Prediction of the CMB

This prediction of the CMB was forgotten because

- it became clear that heavy elements could not have formed in this way because no stable nuclei with A=5,8 exist
- nucleosynthesis in stars became better understood and was able explain the heavy elements
- with heavy elements forming in stars, it was natural to suspect the light elements also formed in stars, even if it was not yet understood how

Prediction of the CMB

The irony is that there was evidence for radiation at a few K from 1941

MOLECULAR LINES FROM THE LOWEST STATES OF DIATOMIC MOLECULES COMPOSED OF ATOMS PROBABLY PRESENT IN INTERSTELLAR SPACE

BY ANDREW McKELLAR

Thus from (3) we find, for the region of space where the CN absorption takes place, the "rotational" temperature,

$$T = 2\overset{\circ}{.}3 K.$$

Discovery of the CMB

Dicke 1964:

> Could a bounce set up a "fireball", a universe filled with hot and dense radiation left over and detectable today?

Discovery of the CMB

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Jim Peebles working on the theory

Discovery of the CMB

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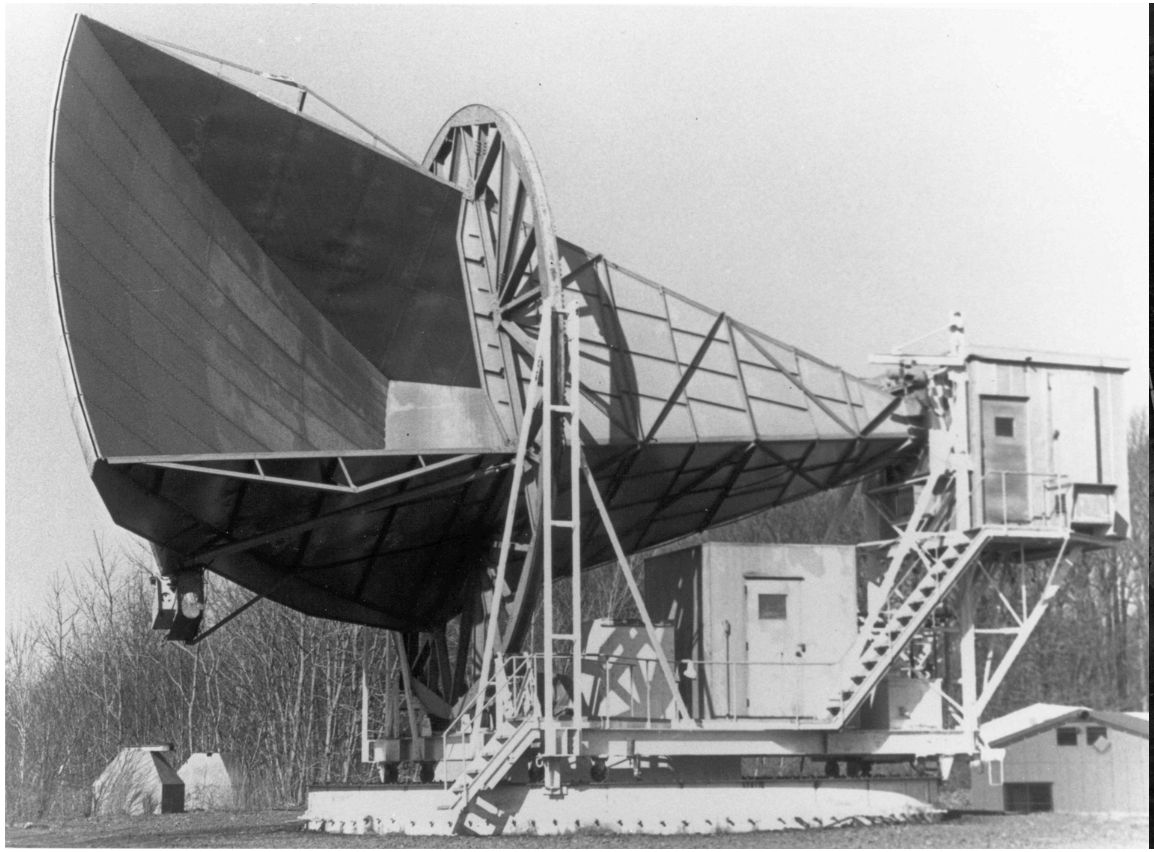
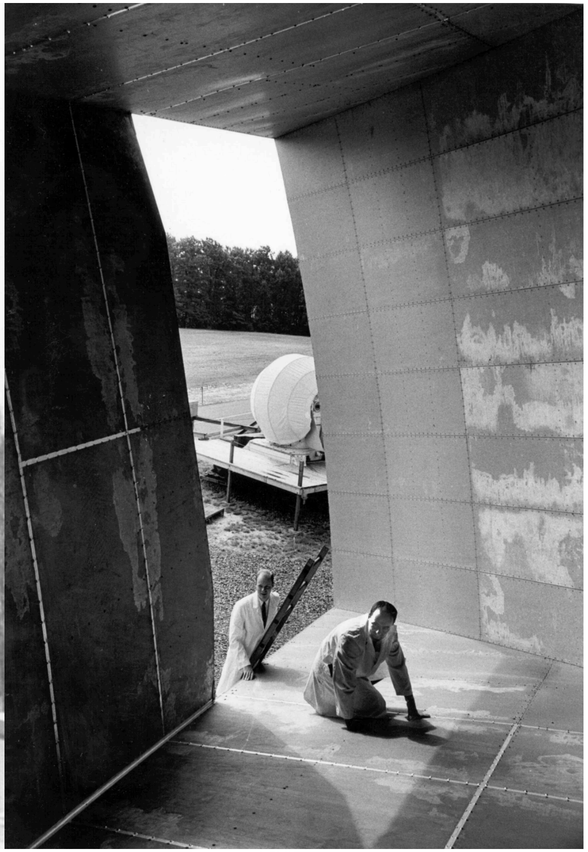
>
> Jim Peebles working on the theory

Roll and Wilkinson with the microwave radiometer

Discovery of the CMB

Meanwhile 30 miles away:





Penzias and Wilson are troubled by noise in their experiment

Discovery of the CMB

Penzias and Wilson are informed by Bernie Burke who is informed by Ken Turner of a talk given by Jim Peebles

COSMIC BLACK-BODY RADIATION*

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

May 7, 1965
Palmer Physical Laboratory
Princeton, New Jersey

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

A. A. Penzias
R. W. Wilson

May 13, 1965
Bell Telephone Laboratories, Inc
Crawford Hill, Holmdel, New Jersey

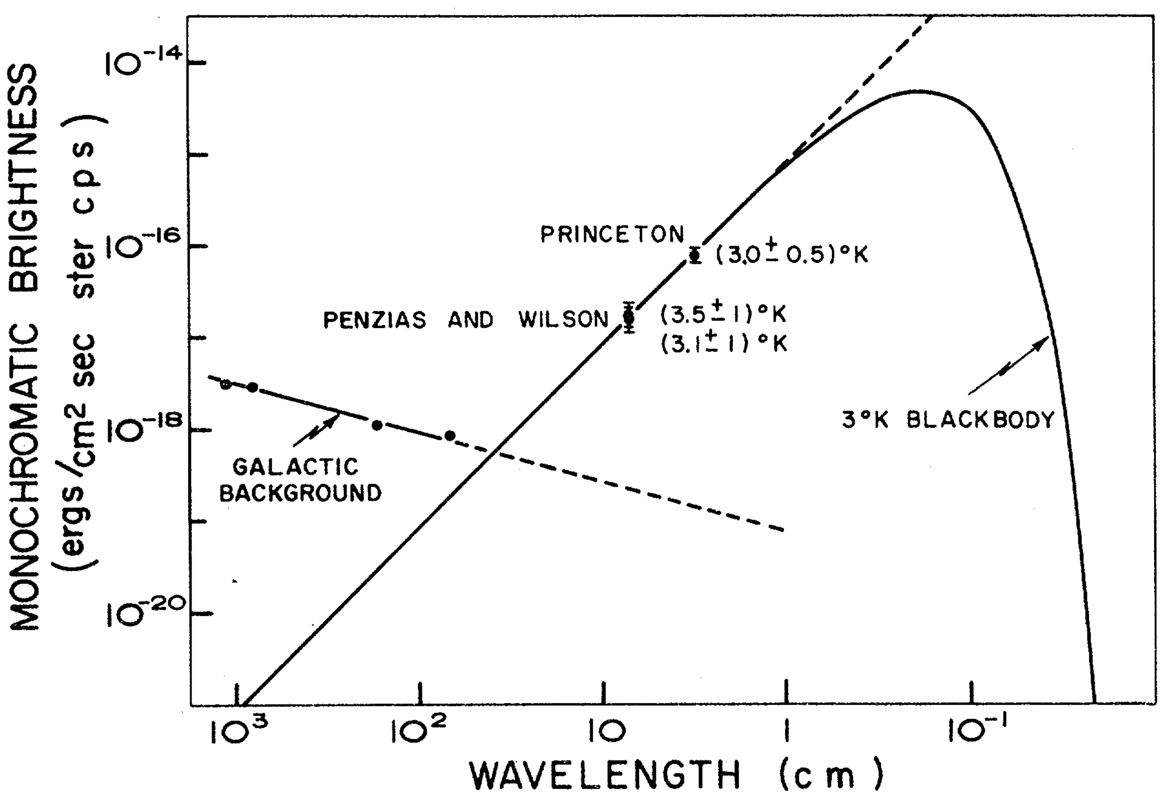
Discovery of the CMB

Additional measurements are required to confirm the interpretation

COSMIC BACKGROUND RADIATION AT 3.2 cm—SUPPORT FOR COSMIC BLACK-BODY RADIATION*

P. G. Roll† and David T. Wilkinson

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received 27 January 1966)



Discovery of the CMB

COSMOLOGICAL BACKGROUND RADIATION SATELLITE

J. Mather
P. Thaddeus
Goddard Institute for Space Studies

R. Weiss
D. Muehlner
Massachusetts Institute of Technology

D. T. Wilkinson
Princeton University

M. G. Hauser
R. F. Silverberg
Goddard Space Flight Center

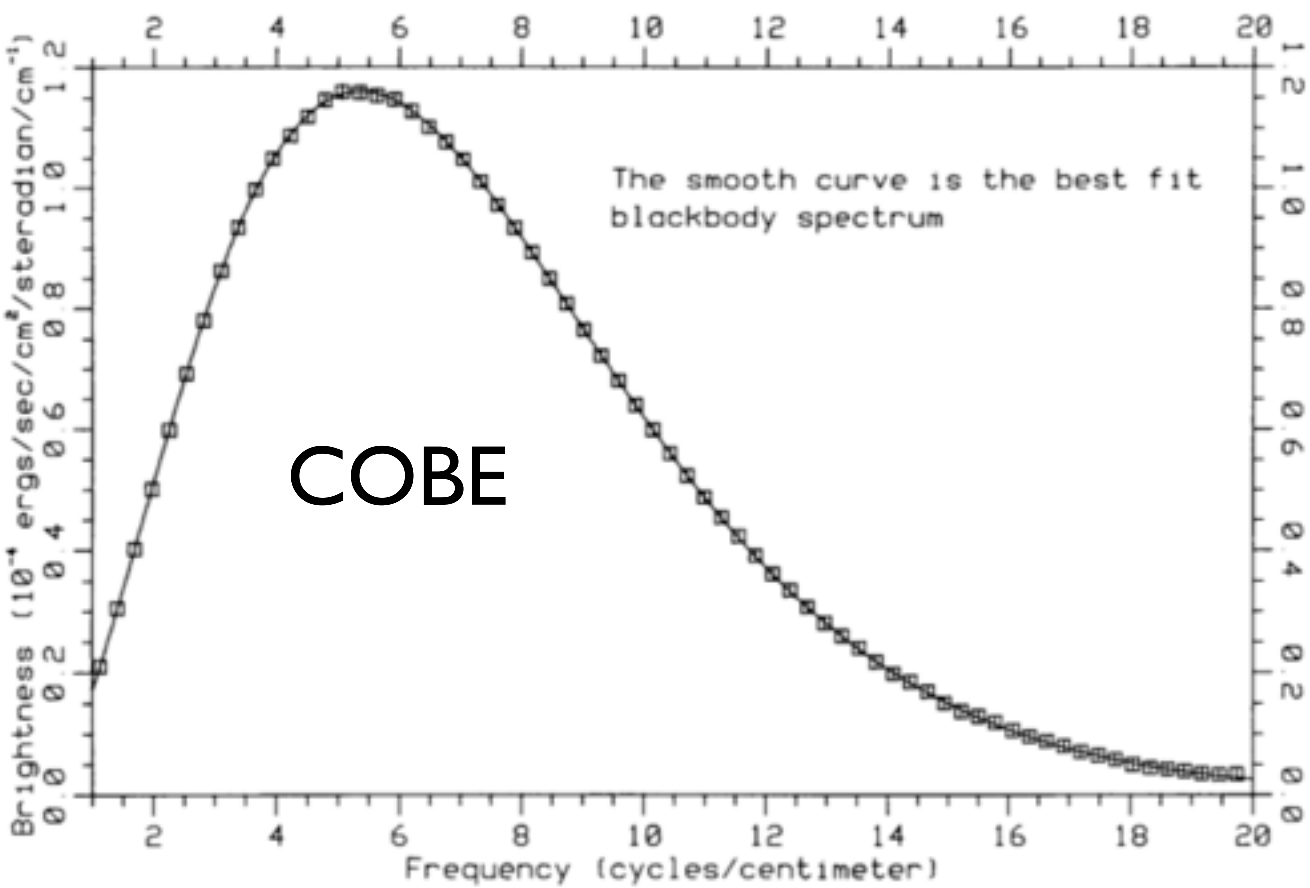
OCTOBER 1974

Discovery of the CMB

A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE *COSMIC BACKGROUND EXPLORER* (*COBE*)[1] SATELLITE

J. C. Mather,[2] E. S. Cheng,[2] R. E. Eplee, Jr.,[3] R. B. Isaacman,[3] S. S. Meyer,[4] R. A. Shafer,[2] R. Weiss,[4] E. L. Wright,[5] C. L. Bennett, N. W. Boggess,[2] E. Dwek,[2] S. Gulkis,[6] M. G. Hauser,[2] M. Janssen,[6] T. Kelsall,[2] P. M. Lubin,[7] S. H. Moseley, Jr.,[2] T. L. Murdock,[8] R. F. Silverberg,[2] G. F. Smoot,[9] and D. T. Wilkinson[10]

Received 1990 January 16; accepted 1990 February 19



Discovery of the CMB

An Attempt to Measure the Far Infrared Spectrum of the Cosmic Background Radiation

H. P. Gush

Department of Physics, University of British Columbia, Vancouver, British Columbia

Received August 13, 1973

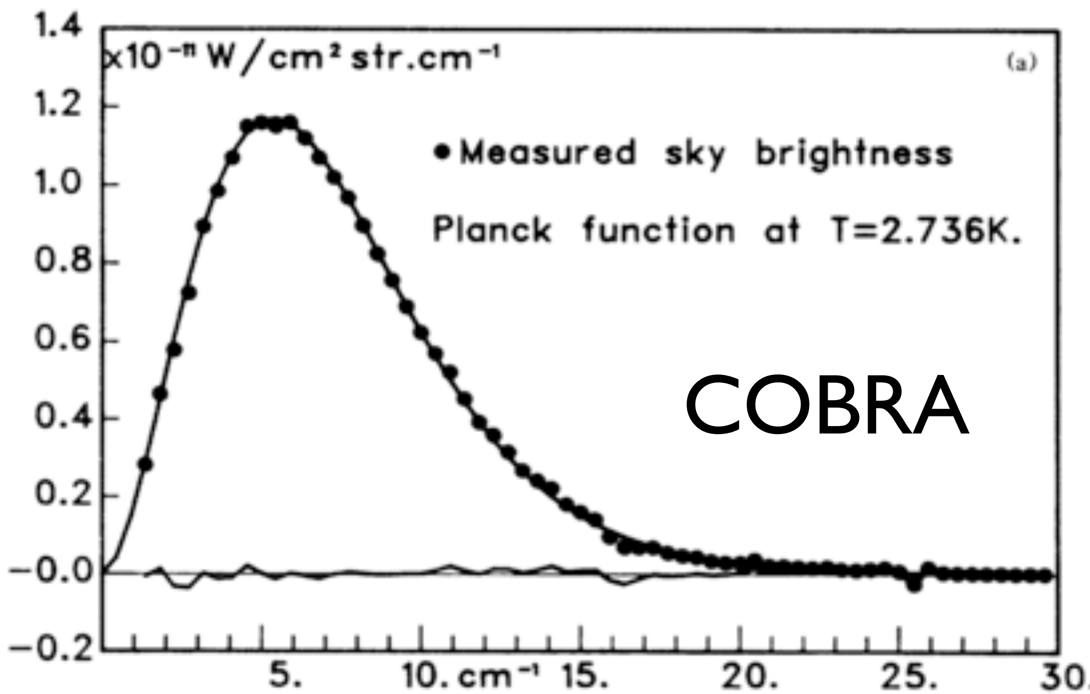
A liquid helium cooled two-beam far infrared interferometer has been successfully flown in a Black Brant III B rocket. The detector was a germanium bolometer cooled to a temperature of 0.37 K by a liquid He^3 refrigerator. The sensitive range was between approximately 5 and 50 cm^{-1}. Satisfactory cosmic spectra were not obtained because of contamination by radiation from the earth.

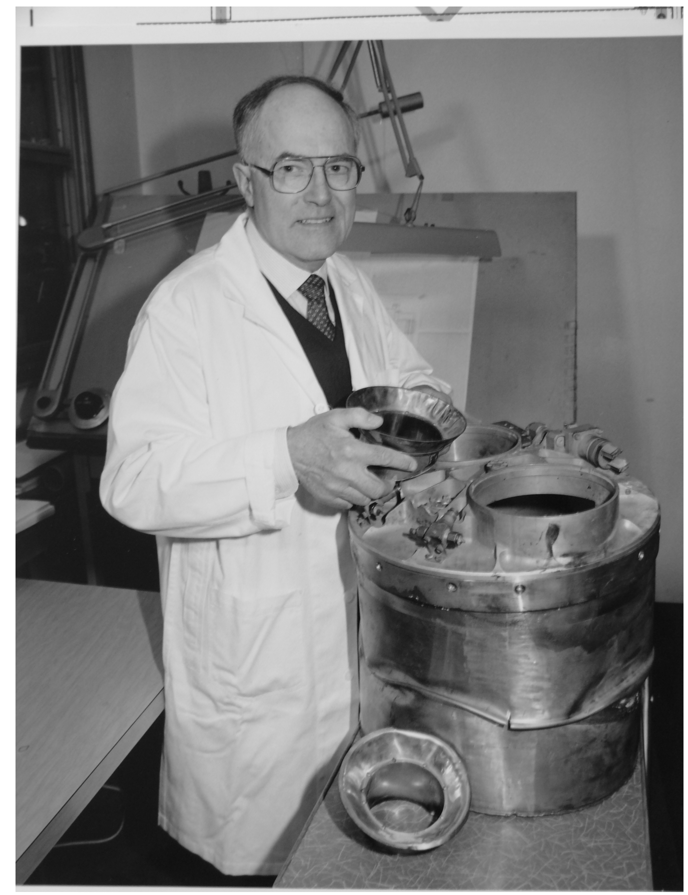
Discovery of the CMB

VOLUME 65, NUMBER 5 PHYSICAL REVIEW LETTERS 30 JULY 1990

Rocket Measurement of the Cosmic-Background-Radiation mm-Wave Spectrum

H. P. Gush, M. Halpern, and E. H. Wishnow
Department of Physics, University of British Columbia, Vancouver, Canada V6T 2A6
(Received 10 May 1990)





Spectrum of the CMB

At early times, (mostly)

Compton scattering	$e^- + \gamma \to e^- + \gamma$
Double Compton scattering	$e^- + \gamma \to e^- + \gamma + \gamma'$
Bremsstrahlung	$e^- + e^- \to e^- + e^- + \gamma$

keep matter and radiation in thermal equilibrium and lead to a black body spectrum for the photons.

$$n_{T(t)}(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{\exp\left(h\nu/kT(t)\right) - 1}$$

Spectrum of the CMB

At some point radiation no longer efficiently scatters off matter and thermal equilibrium is no longer maintained.

So (why) do we expect to observe a black body spectrum today?

Consider an idealization:

- All photons last scatter at same time
- Black body spectrum until last scattering
- Ignore processes that inject photons

Spectrum of the CMB

Or put differently, how does the expansion affect the spectrum

$$n_{T(t)}(\nu)d\nu = \left(\frac{a(t_L)}{a(t)}\right)^3 n_{T(t_L)}\left(\nu a(t)/a(t_L)\right) d\left(\nu a(t)/a(t_L)\right)$$

or

$$n_{T(t)}(\nu)d\nu = \frac{8\pi\nu^2 d\nu}{\exp\left(h\nu/kT(t)\right) - 1}$$

with

$$T(t) = T(t_L)\frac{a(t_L)}{a(t)}$$

For massless quanta the expansion preserves a black body distribution after last scattering

Spectrum of the CMB

This remains true if last scattering is not instantaneous provided scattering events around last scattering do not change the photon energies

When does last scattering occur?

Photons will scatter efficiently as long as

$$n_e \sigma_T c \gtrsim H$$

Spectrum of the CMB

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When does last scattering occur?

Photons will scatter efficiently as long as

$$n_e \sigma_T c \gtrsim H$$

If there were no recombination and

$$n_e \approx n_b = \frac{\rho_{b,0}}{m_p} \left(\frac{a(t_0)}{a(t)} \right)^3$$

this would happen at temperatures around 100K.

Spectrum of the CMB

When does (re)combination occur?

In thermal equilibrium

$$\frac{n_{1s}}{n_e n_p} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(B/kT\right)$$

Neutrality implies $n_e = n_p$ (after Helium recombination)

The free electron fraction $x_e = \dfrac{n_e}{n_p + n_{1s}}$

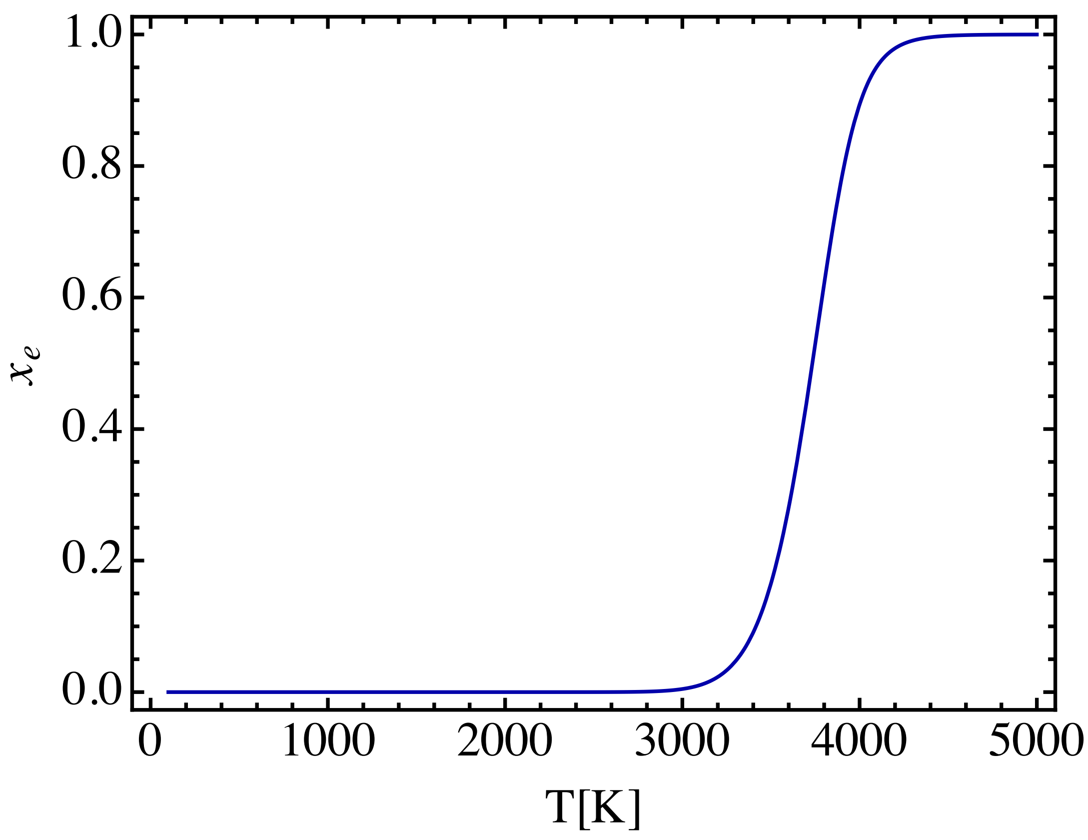
then satisfies the Saha equation

$$\frac{1-x_e}{x_e^2} = (1 - Y_{He}) n_b \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-3/2} \exp\left(B/kT\right)$$

Spectrum of the CMB

When does (re)combination occur?

In thermal equilibrium between 3000K and 4000K



Spectrum of the CMB

However, recombination occurs out of equilibrium

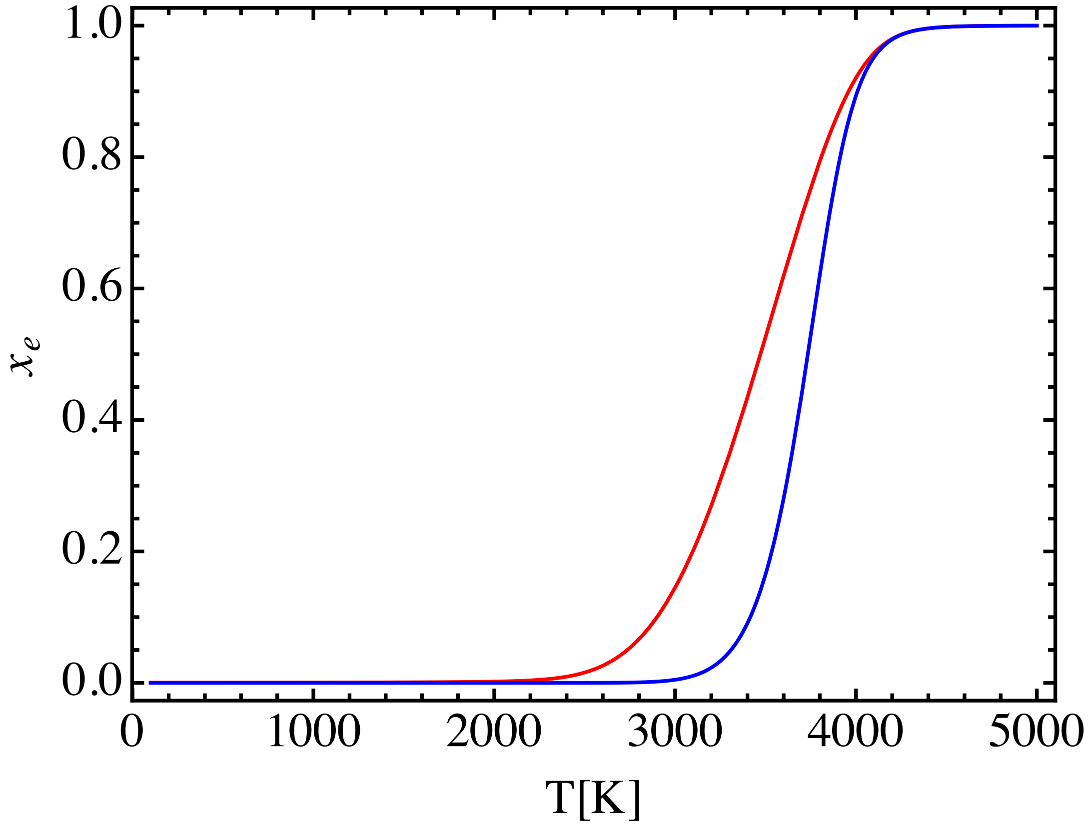
- photons emitted when electrons are captured into low lying energy levels ionize other atoms
- photons emitted in transitions from highly excited states to low lying states excite other atoms
- Ly-α photons excite other atoms from the ground state, making $2p \rightarrow 1s$ recombination inefficient so that $2s \rightarrow 1s$ is relevant

Peebles and independently Zel'dovich, Kurt, Sunyaev in 1968 derived

$$\frac{dx_e}{dt} = -C\left[\alpha n_p x_e - (1 - x_e)\beta \exp(E_{12}/kT)\right]$$

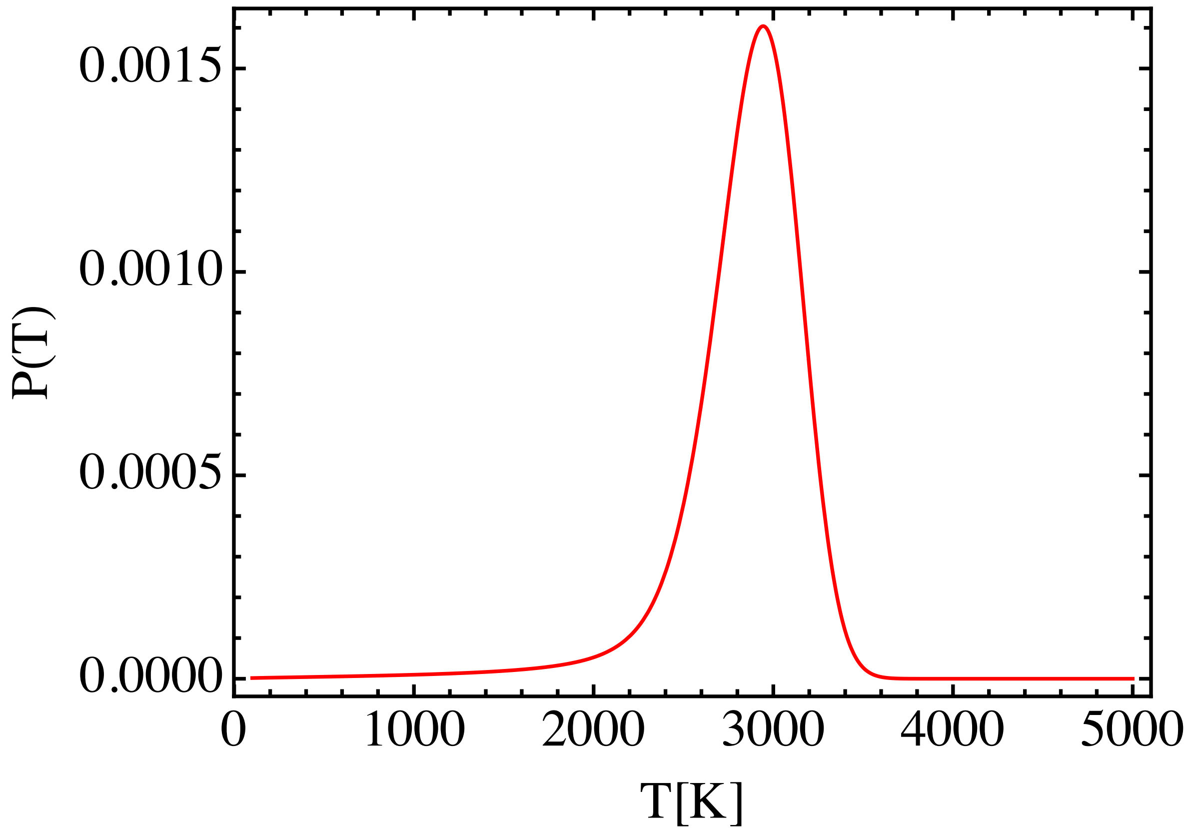
Spectrum of the CMB

Including departures from equilibrium delay recombination



Spectrum of the CMB

Last scattering probability peaks near 3000K



Spectrum of the CMB

So photons last scatter around 3000K.
Is energy still exchanged efficiently then?

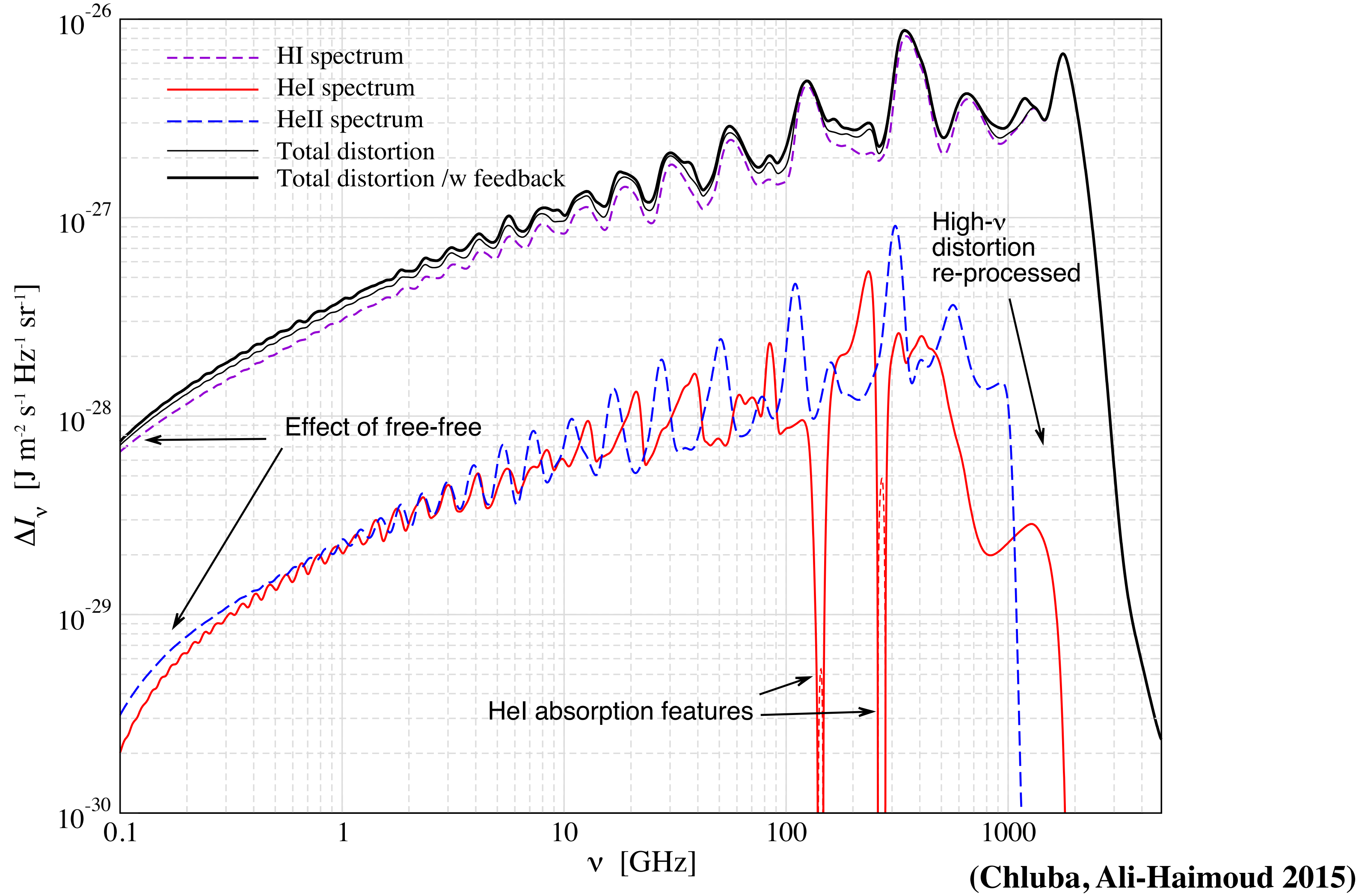
$$\frac{kT}{m_e} n_e \sigma_T c < H \quad \text{below } 10^5 K$$

Thomson scattering can only modify the spectrum at temperatures above $10^5 K$, not around last scattering.

So the spectrum is preserved even if not all photons last scatter at the same instant.

Spectrum of the CMB

However, if a process injects photons around recombination, we expect small spectral distortions



(Chluba, Ali-Haimoud 2015)

Spectrum of the CMB

Above $10^5 K$ energy is exchanged efficiently, but until when are photons efficiently produced?

Double Compton scattering is inefficient when

$$\alpha \left(\frac{kT}{m_e}\right)^2 n_e \sigma_T c < H$$ i.e. below $6 \times 10^6 K$

So

$$T > 6 \times 10^6 K$$ black body

$$10^5 K < T < 6 \times 10^6 K$$ μ-era

$$T < 10^5 K$$ y-era

Spectrum of the CMB

Spectral distortions from reionization

Interactions of photons with hot electrons from reionization is described by the Kompaneets equation

$$\frac{\partial N_\gamma(\omega)}{\partial t} = \frac{n_e \sigma_T}{m_e \omega^2} \frac{\partial}{\partial \omega} \left[k T_e \omega^4 \frac{\partial N_\gamma(\omega)}{\partial \omega} + \omega^4 N_\gamma(\omega)(1 + N_\gamma(\omega)) \right]$$

For small distortions of the black body spectrum

$$N_\gamma(\omega) = \overline{N}_\gamma(\omega) + \Delta N_\gamma(\omega)$$

this becomes

$$\frac{\partial \Delta N_\gamma(\omega)}{\partial t} = \frac{n_e \sigma_T}{m_e \omega^2} \frac{\partial}{\partial \omega} \left[k(T_e - T) \omega^4 \frac{\partial \overline{N}_\gamma(\omega)}{\partial \omega} \right]$$

Spectrum of the CMB

The spectral distortion today is then given by

$$\Delta N_\gamma(\omega) = \int_0^{t_0} dt' \, \frac{n_e \sigma_T k(T_e - T)}{m_e} \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \frac{\partial \overline{N}_\gamma(\omega)}{\partial \omega} \right]$$

or more compactly

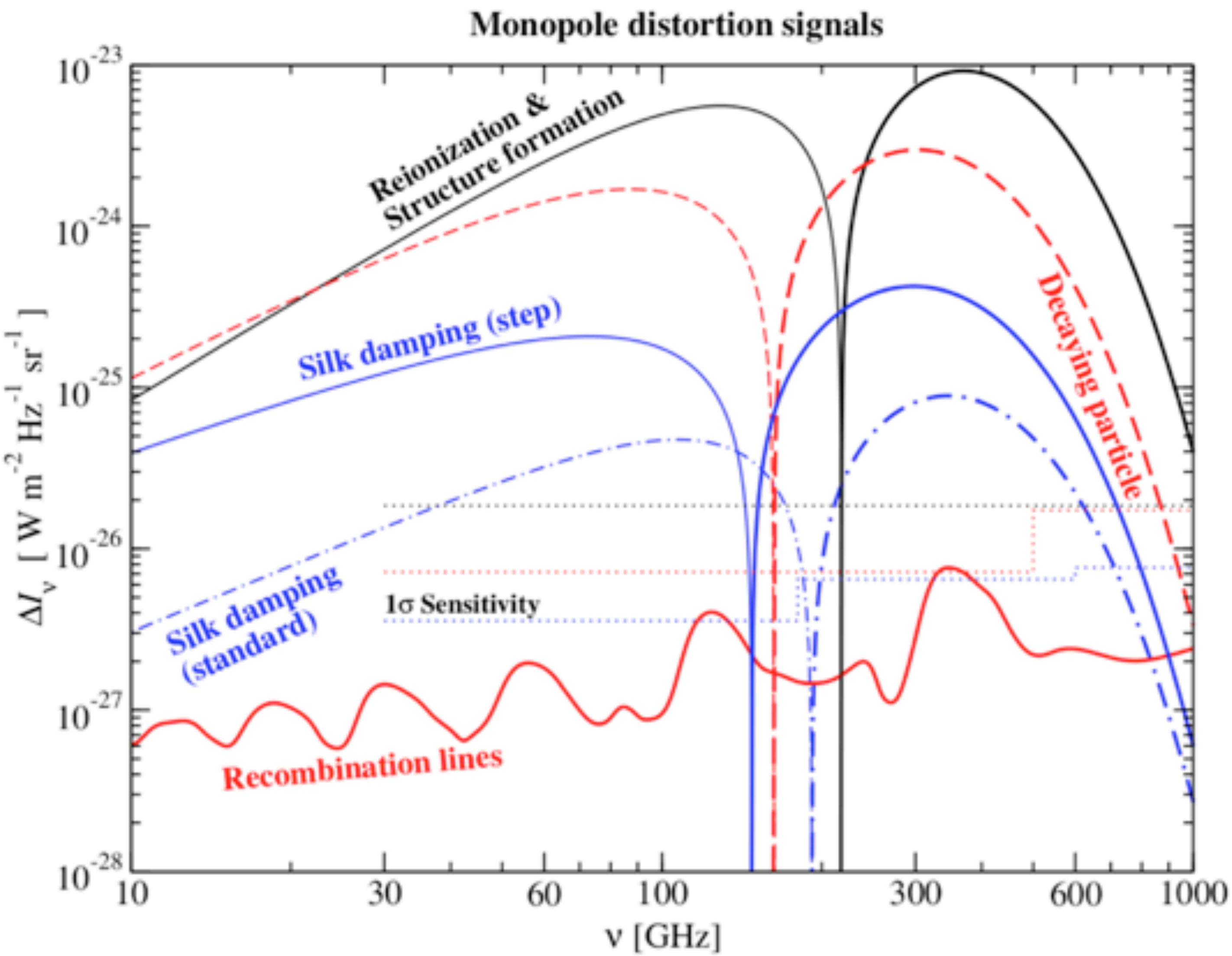
$$\Delta N_\gamma(\omega) = y \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \frac{\partial \overline{N}_\gamma(\omega)}{\partial \omega} \right]$$

With the Compton-y parameter

$$y = \int_0^{t_0} dt' \, \frac{n_e \sigma_T k(T_e - T)}{m_e}$$

Spectrum of the CMB

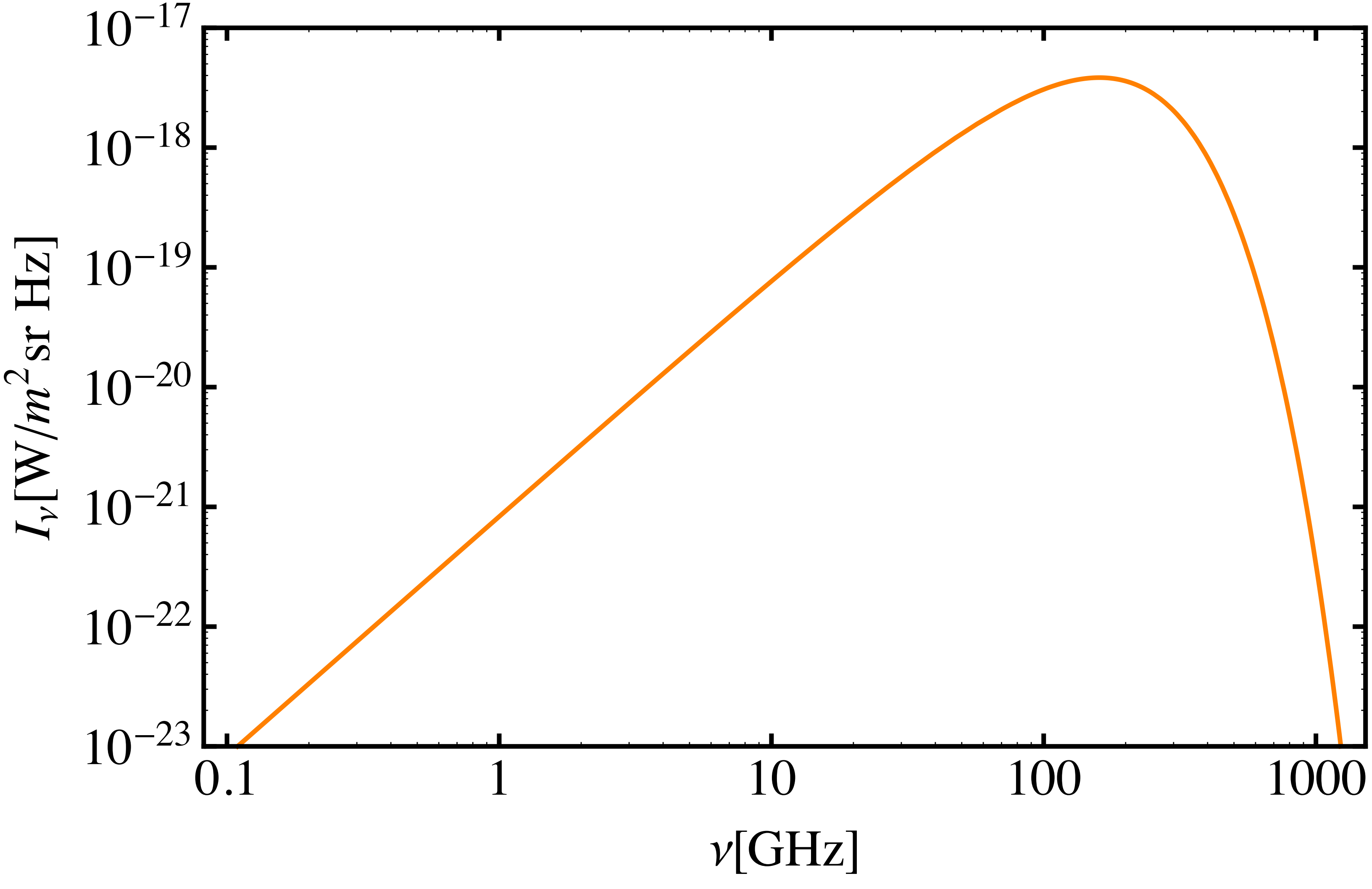
Spectral distortions



(Andre et al. 2013)

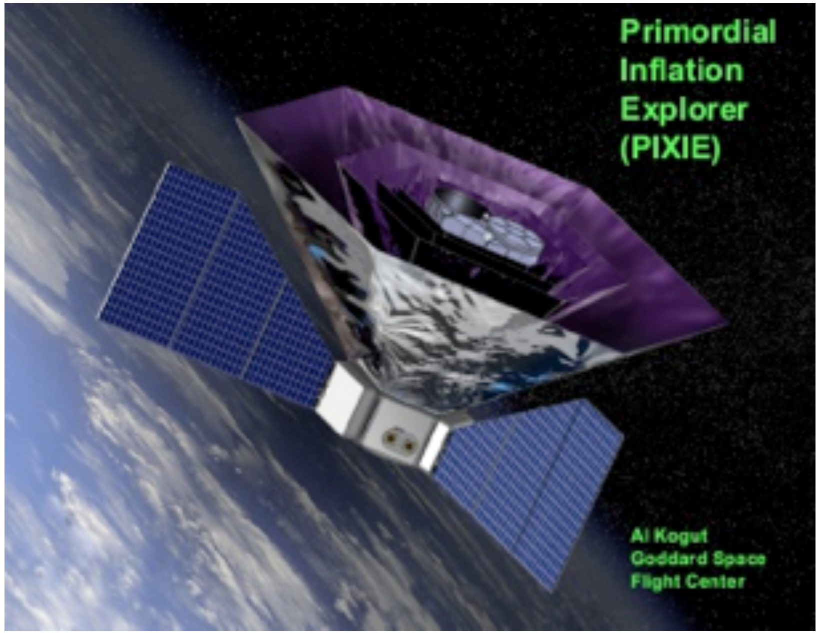
Spectrum of the CMB

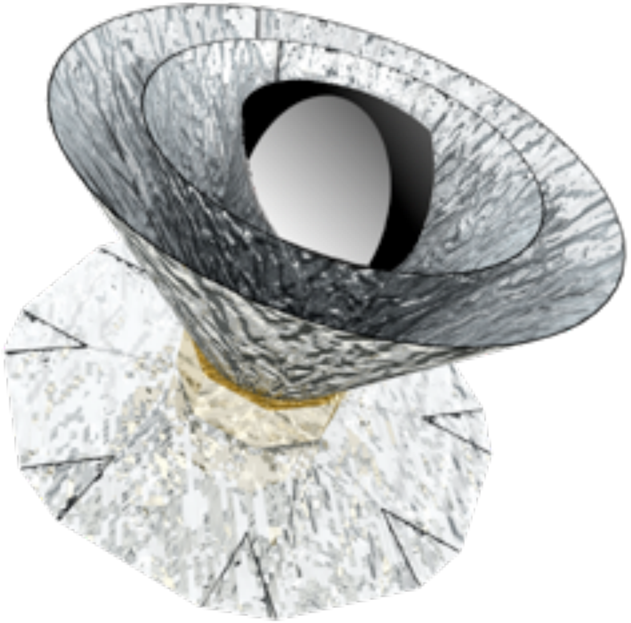
Note the scale



Spectrum of the CMB

Rather remarkably, potential small distortions may be detectable in future experiments





PRISM

(Neither was funded but hopefully some such experiment will be)

Thermal History

We now know the behavior of the universe to fairly high redshifts or early times

0<z<0.3	dark energy	$\rho \approx \rho_0 \Omega_\Lambda$
0.3<z<3300	matter	$\rho = \rho_0 \Omega_m \left(\frac{a_0}{a}\right)^3$
3300<z<?	radiation	$\rho = \rho_0 \Omega_r \left(\frac{a_0}{a}\right)^4$

This holds until temperatures become so high that e^+e^- pairs form

$$T \simeq 511\mathrm{keV} \simeq 6 \times 10^9 \mathrm{K}$$

Thermal History

Weak and electromagnetic interactions rapidly thermalize the universe at early times and

$$\rho(T, \mu_e, \mu_p, \mu_\nu, ...)$$

$$p(T, \mu_e, \mu_p, \mu_\nu, ...)$$

$$s(T, \mu_e, \mu_p, \mu_\nu, ...)$$

We know chemical potentials for electrons and protons are small, if we assume chemical potentials for neutrinos are negligible as well

$$\rho(T) \qquad\qquad p(T) \qquad\qquad s(T)$$

Thermal History

The first law of thermodynamics

$$dU = TdS - pdV$$

then leads us to

$$s = \frac{p + \rho}{T}$$

$$\frac{dp}{dT} = \frac{p + \rho}{T}$$

In addition, for adiabatic processes

$$sa^3 = \text{const}$$

Thermal History

To close the system of equations, we need equation of state. For relativistic particles

$$p = \frac{1}{3}\rho$$

So

$$\frac{d\rho}{dT} = \frac{4\rho}{T}$$

leads to

$$\rho(T) = \alpha T^4$$

$$p(T) = \frac{1}{3}\alpha T^4$$

$$s(T) = \frac{4}{3}\alpha T^3$$

Thermal History

To determine the integration constant α , we must return to the microscopic description. For relativistic particles

$$\rho(T) = g \int \frac{d^3 p}{(2\pi)^3} \frac{p}{e^{p/kT} \pm 1}$$

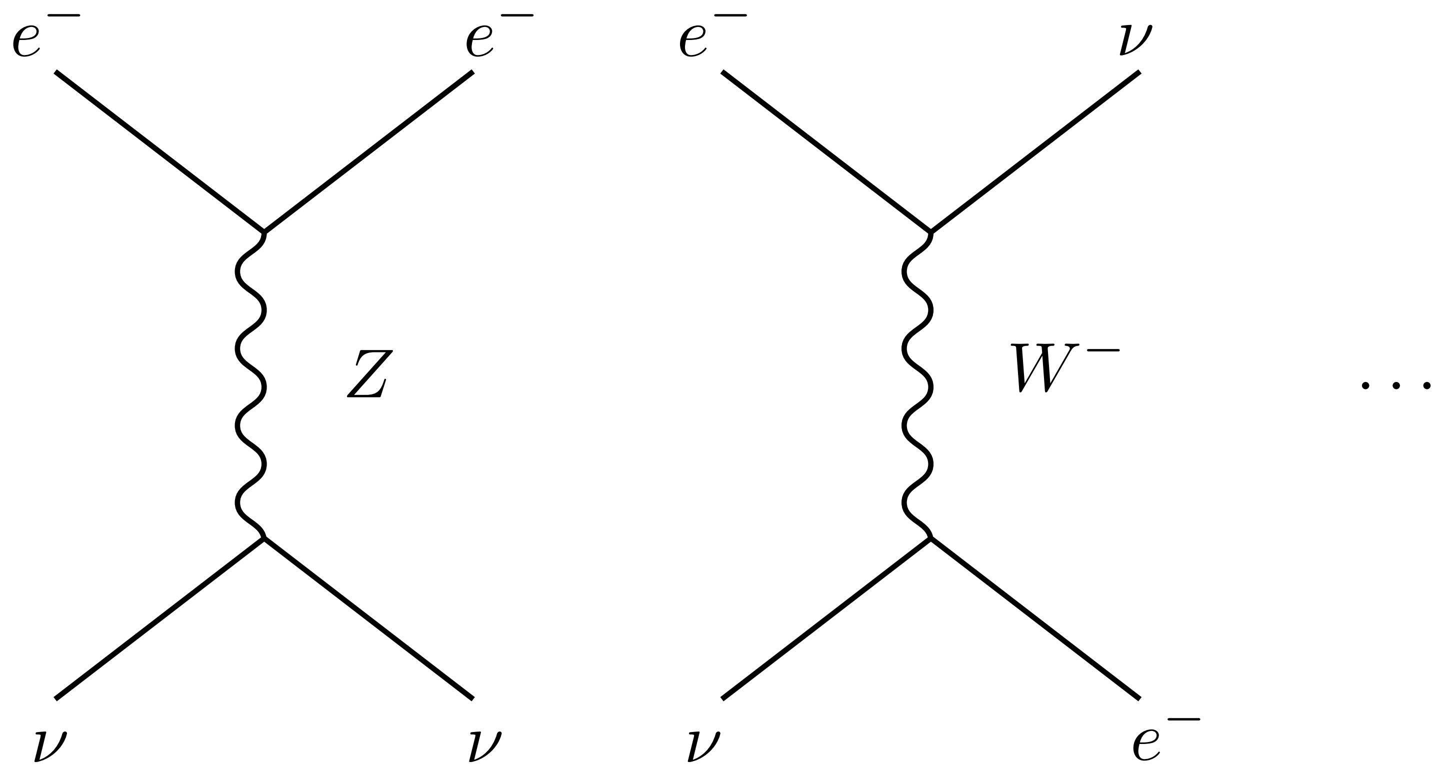
$$\rho(T) = g \frac{\pi^2}{30} (kT)^4 \times \begin{cases} 7/8 & \text{fermions} \\ 1 & \text{bosons} \end{cases}$$

For example, when γ, e^+, e^-, ν are in thermal equilibrium

$$\rho(T) = \frac{\pi^2}{30} (kT)^4 \left(2 + \frac{7}{8} (2 \times 2 + 3 \times 2) \right) = \frac{\pi^2}{30} (kT)^4 \frac{43}{4}$$

Thermal History

Neutrinos are kept in equilibrium through the weak interactions



$$\Gamma \sim G_F^2 T^5$$

Thermal History

With

$$\Gamma \sim G_F^2 T^5$$

and

$$H \simeq \sqrt{\frac{8\pi G}{3}\frac{\pi^3}{30}\frac{43}{4}T^4}$$

kinetic decoupling occurs when $T \sim 1\,\mathrm{MeV}$, before and around the time e^+e^- annihilate.

After kinetic decoupling, $p \propto a^{-1}$ and $T_\nu \propto a^{-1}$

$$s_\nu a^3 \to \mathrm{const}$$

Thermal History

Since the total comoving entropy is conserved, the entropy stored in e^+e^- must then be transferred to photons.

$$(s_\gamma + s_e)a^3\big|_{\text{before}} = s_\gamma a^3\big|_{\text{after}}$$

$$\left(2 + \frac{7}{8} \times 2 \times 2\right) T^3_{\text{before}} = 2T^3_{\text{after}}$$

We can write this as

$$T_\gamma = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

Thermal History

So if neutrinos were completely decoupled when e^+e^- annihilate, the energy density would be

$$\rho(T) = \frac{\pi^2}{15}(kT_\gamma)^4 \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \times 3\right)$$

Taking into account QED corrections and that decoupling is not quite complete

$$\rho(T) = \frac{\pi^2}{15}(kT_\gamma)^4 \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right)$$

with $N_{\mathrm{eff}} = 3.046$ in the Standard Model

Nucleosynthesis

Equilibrium abundances

If Z_i protons and A_i-Z_i neutrons can rapidly form a nucleus of type i, its chemical potential must be

$$\mu_i = Z_i \mu_p + (A_i - Z_i)\mu_n$$

and the equilibrium abundance of nuclei of type i is

$$n_i = g_i \left(\frac{m_i kT}{2\pi\hbar^2} \right)^{3/2} e^{-\mu_i/kT} e^{-m_i/kT}$$

Nucleosynthesis

The chemical potential is typically unknown, but we can compute ratios that are independent of μ_i

$$\frac{n_i}{n_p^{Z_i} n_n^{A_i - Z_i}} = \frac{g_i}{2^{A_i}} A_i^{3/2} \left(\frac{2\pi\hbar^2}{m_p kT} \right)^{3/2(A_i - 1)} e^{B_i/kT}$$

where

$$B_i = Z_i m_p + (A_i - Z_i) m_n - m_i$$

is the binding energy.

Nucleosynthesis

Introducing

$$X_i = \frac{n_i}{n_b}$$

This becomes

$$X_i = \frac{g_i}{2^{A_i}} A_i^{3/2} \epsilon^{A_i - 1} X_p^{Z_i} X_n^{A_i - Z_i} e^{B_i/kT}$$

with

$$\epsilon = n_b \left(\frac{2\pi\hbar^2}{m_p kT} \right)^{3/2} \propto \frac{n_b}{n_\gamma} \left(\frac{T}{m_p} \right)^{3/2}$$

Nucleosynthesis

So nuclei of type i are rare until

$$T = T_i \simeq \frac{B_i/k}{(A_i - 1)|\ln \epsilon|}$$

The small baryon-to-photon ratio lowers the temperature at which nuclei become abundant.

In equilibrium, nuclei with higher binding energy per nucleon become abundant at higher temperatures

$$B_d = 2.2\mathrm{MeV}$$

$$B_{He} = 28.3\mathrm{MeV}$$

i.e. Helium appears at higher temperatures than deuterium

Nucleosynthesis

Beyond equilibrium

After the QCD phase transition, the universe is filled with

$$e^- \qquad \nu$$

$$\gamma \qquad\qquad\qquad p$$

$$e^+ \qquad n$$

and densities are too low for many-body processes.

Helium can only form after deuterium forms and nucleosynthesis must occurs out of equilibrium.

Nucleosynthesis

Neutron abundance

Neutrons and protons are can be converted into each other through weak interactions

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

$$n + \nu_e \leftrightarrow p + e^-$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

So

$$\frac{d(a^3 n_n)}{dt} = -\lambda_{np} a^3 n_n + \lambda_{pn} a^3 n_p$$

or

$$\frac{dX_n}{dt} = -\lambda_{np} X_n + \lambda_{pn}(1 - X_n)$$

Nucleosynthesis

The rates are not independent. For the right hand side to vanish in thermal equilibrium

$$\lambda_{np} X_n^{eq} = \lambda_{pn}(1 - X_n^{eq})$$

and from our equilibrium considerations we know

$$\frac{X_n^{eq}}{1 - X_n^{eq}} = e^{-Q/kT}$$

with

$$Q = m_n - m_p = 1.293\,\mathrm{MeV}$$

So

$$\frac{dX_n}{dt} = -\lambda_{np}(1 + e^{-Q/kT})(X_n - X_n^{eq})$$

Nucleosynthesis

The rates can be calculated in quantum field theory, and the equation can readily be solved numerically.

Until the formation of nuclei

$$X_n \approx 0.16\, e^{-t/\tau_n}$$

Interpretation

$t \ll \tau_n$ decays negligible

two-body processes active $X_n \approx X_n^{eq}$

two-body processes inefficient $X_n \approx \mathrm{const}$

$t \gg \tau_n$ two-body processes negligible
only decays important

Nucleosynthesis

Deuterium formation

Collisions of neutrons and protons form deuterium

$$p + n \leftrightarrow d + \gamma$$

Occurs rapidly and deuterium abundance is well approximated by equilibrium value

$$X_d = \frac{3}{\sqrt{2}} \epsilon X_p X_n e^{B_d/kT}$$

Photodissociation keeps deuterium abundance low

Nucleosynthesis

<u>Heavier elements</u>

Nucleosynthesis begins when photo-dissociation becomes inefficient enough for deuterons to capture additional neutrons or collisions of deuterons to form tritium and helium.

$$d + d \to {}^3H + p$$

$$d + d \to {}^3He + n$$

$$d + p \to {}^3He + \gamma$$

(suppressed)

$$d + n \to {}^3H + \gamma$$

Nucleosynthesis

Once these interactions become efficient, Helium rapidly forms

$$d + {}^3H \to {}^4He + n$$

$$ {}^3He + n \to {}^3H + p$$

$$d + {}^3He \to {}^4He + p$$

so the Helium mass fraction Y_{He} is

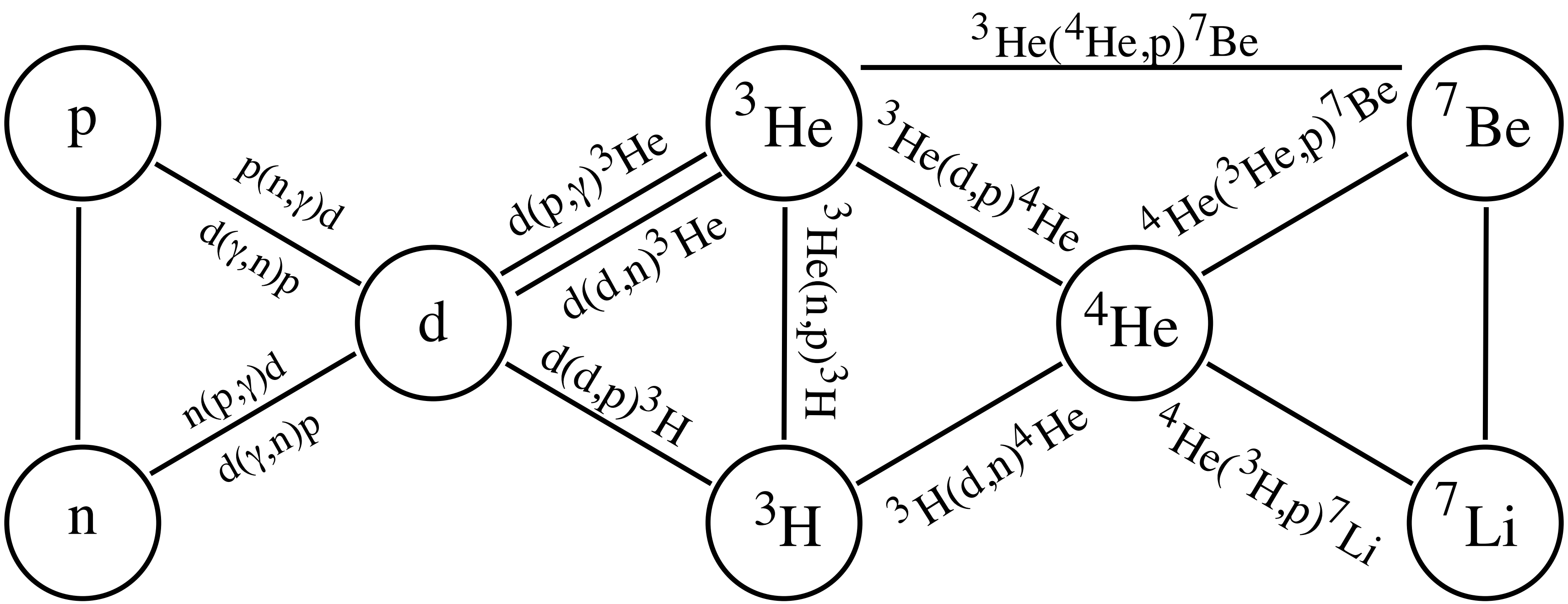
$$Y_{He} = \frac{4n_{He}}{n_N + 4n_{He}} = \frac{2n_n}{n_b} = 2X_n$$

or

$$Y_{He} \approx 0.16\, e^{-t_d/\tau_n} \approx 0.25$$

Nucleosynthesis

To go further, we must consider a larger network of nuclear interactions.



This is usually done numerically.

Nucleosynthesis

The more detailed numerical work was done by Fermi and Turkevich (but not published).

No.	Reaction	Specific reaction rates	Term in rate equations, $\mathcal{R}'$ [See Eq. (132)]
1	$N=H+e^-$	10^{-3} sec.$^{-1}$	$10^{-3}\mathbf{x}_N$
2	$N+H=D+h\nu$	6.6×10^{-20} sec.$^{-1}$	$6.6\times10^{-20}q_0\mathbf{x}_N\mathbf{x}_H t^{-3/2}$
3	$N+D=T+h\nu$	2.0×10^{-22} sec.$^{-1}$	$2.0\times10^{-22}q_0\mathbf{x}_N\mathbf{x}_D t^{-3/2}$
4	$N+D=N+N+H$	Negligible (see reaction 18)	0
5	$N+He^3=He^4+h\nu$	10^{-21} sec.$^{-1}$ (estimated)	$10^{-21}q_0\mathbf{x}_N\mathbf{x}_{He^3}t^{-3/2}$
6	$N+He^3=T+H$	1.5×10^{-15} sec.$^{-1}$	$1.5\times10^{-15}q_0\mathbf{x}_N\mathbf{x}_{He^3}t^{-3/2}$
7	$H+H=D+e^+$	$a_1=2\times10^{-39}$; $a_2=3.16$	$7.0\times10^{-41}q_0(\mathbf{x}_H)^2t^{-7/6}10^{-0.592t^{1/6}}$
8	$H+D=He^3+h\nu$	$a_1=8.6\times10^{-21}$; $a_2=3.48$	$3.0\times10^{-22}q_0\mathbf{x}_H\mathbf{x}_D t^{-7/6}10^{-0.652t^{1/6}}$
9	$H+D=H+H+N$	Negligible (see reaction 18)	0
10	$H+T=He^4+h\nu$	$a_1=1.5\times10^{-19}$; $a_2=3.62$	$5.3\times10^{-21}q_0\mathbf{x}_H\mathbf{x}_T t^{-7/6}10^{-0.678t^{1/6}}$
11	$H+T=He^3+N$	$1.5\times10^{-15}\times10^{-36.8/T_8}$ sec.$^{-1}$	$1.5\times10^{-15}q_0\mathbf{x}_H\mathbf{x}_T t^{-3/2}10^{-0.242t^{1/2}}$
12	$D+D=He^4+h\nu$	$a_1=3.07\times10^{-19}$; $a_2=3.99$	$1.08\times10^{-20}q_0(\mathbf{x}_D)^2t^{-7/6}10^{-0.747t^{1/6}}$
13	$D+D=He^3+N$	$a_1=3.0\times10^{-15}$; $a_2=3.99$	$1.1\times10^{-16}q_0(\mathbf{x}_D)^2t^{-7/6}10^{-0.747t^{1/6}}$
14	$D+D=H+T$	$a_1=3.0\times10^{-15}$; $a_2=3.99$	$1.1\times10^{-16}q_0(\mathbf{x}_D)^2t^{-7/6}10^{-0.747t^{1/6}}$
15	$D+T=He^4+N$	$a_1=5.0\times10^{-13}$; $a_2=4.24$	$1.8\times10^{-14}q_0\mathbf{x}_D\mathbf{x}_T t^{-7/6}10^{-0.794t^{1/6}}$
16	$D+He^3=He^4+H$	$a_1=1.5\times10^{-12}$; $a_2=6.72$	$5.3\times10^{-14}q_0\mathbf{x}_D\mathbf{x}_{He^3}t^{-7/6}10^{-1.259t^{1/6}}$
17	$D+He^4=Li^6+h\nu$	$a_1=1.4\times10^{-21}$; $a_2=6.96$	$4.9\times10^{-23}q_0\mathbf{x}_D\mathbf{x}_{He^4}t^{-7/6}10^{-1.304t^{1/6}}$
18[a]	$D+h\nu=H+N$	$5.9\times10^{12}T_8^{3/2}10^{-110/T_8}$ sec.$^{-1}$	$1.1\times10^{+16}\mathbf{x}_D t^{-3/4}10^{-0.723t^{1/2}}$
19	$T=He^3+e^-$	1.8×10^{-9} sec.$^{-1}$	$1.8\times10^{-9}\mathbf{x}_T$
20	$T+T=He^4+N+N$	$a_1=2.6\times10^{-13}$; $a_2=4.57$	$9.1\times10^{-15}q_0(\mathbf{x}_T)^2t^{-7/6}10^{-0.856t^{1/6}}$
21	$T+T=He^6+h\nu$	$a_1=2.6\times10^{-19}$; $a_2=4.57$	$9.1\times10^{-21}q_0(\mathbf{x}_T)^2t^{-7/6}10^{-0.856t^{1/6}}$
22	$T+He^3=He^4+N+H$	$a_1=1.5\times10^{-12}$; $a_2=7.24$	$5.3\times10^{-14}q_0\mathbf{x}_T\mathbf{x}_{He^3}t^{-7/6}10^{-1.356t^{1/6}}$
23	$T+He^3=He^4+D$	$a_1=1.0\times10^{-13}$; $a_2=7.24$	$3.5\times10^{-15}q_0\mathbf{x}_T\mathbf{x}_{He^3}t^{-7/6}10^{-1.356t^{1/6}}$
24	$T+He^3=Li^6+h\nu$	$a_1=3.1\times10^{-18}$; $a_2=7.24$	$1.1\times10^{-19}q_0\mathbf{x}_T\mathbf{x}_{He^3}t^{-7/6}10^{-1.356t^{1/6}}$
25	$T+He^4=Li^7+h\nu$	$a_1=5.5\times10^{-19}$; $a_2=7.56$	$1.9\times10^{-20}q_0\mathbf{x}_T\mathbf{x}_{He^4}t^{-7/6}10^{-1.416t^{1/6}}$
26	$He^3+He^3=Be^6+h\nu$	$a_1=1.4\times10^{-17}$; $a_0=11.49$	$4.9\times10^{-19}q_0(\mathbf{x}_{He^3})^2t^{-7/6}10^{-2.151t^{1/6}}$
27	$He^3+He^3=He^4+H+H$	$a_1=1.4\times10^{-11}$; $a_2=11.49$	$4.9\times10^{-13}q_0(\mathbf{x}_{He^3})^2t^{-7/6}10^{-2.151t^{1/6}}$
28	$He^3+He^4=Be^7+h\nu$	$a_1=1.7\times10^{-19}$; $a_2=12.01$	$6.0\times10^{-21}q_0\mathbf{x}_{He^3}\mathbf{x}_{He^4}t^{-7/6}10^{-2.250t^{1/6}}$

[a] The photon concentration is included in the constant.

(used wrong initial conditions)

Nucleosynthesis

At the time the work by Alpher, Gamow, Fermi and others and their prediction of the CMB was forgotten because

- it became clear that heavy elements could not have formed in this way because no stable nuclei with A=5,8 exist
- nucleosynthesis in stars became better understood and was able explain the heavy elements

Nucleosynthesis

Hoyle 1964:

Nucleosynthesis in stars can explain abundances of heavy elements, but not of helium

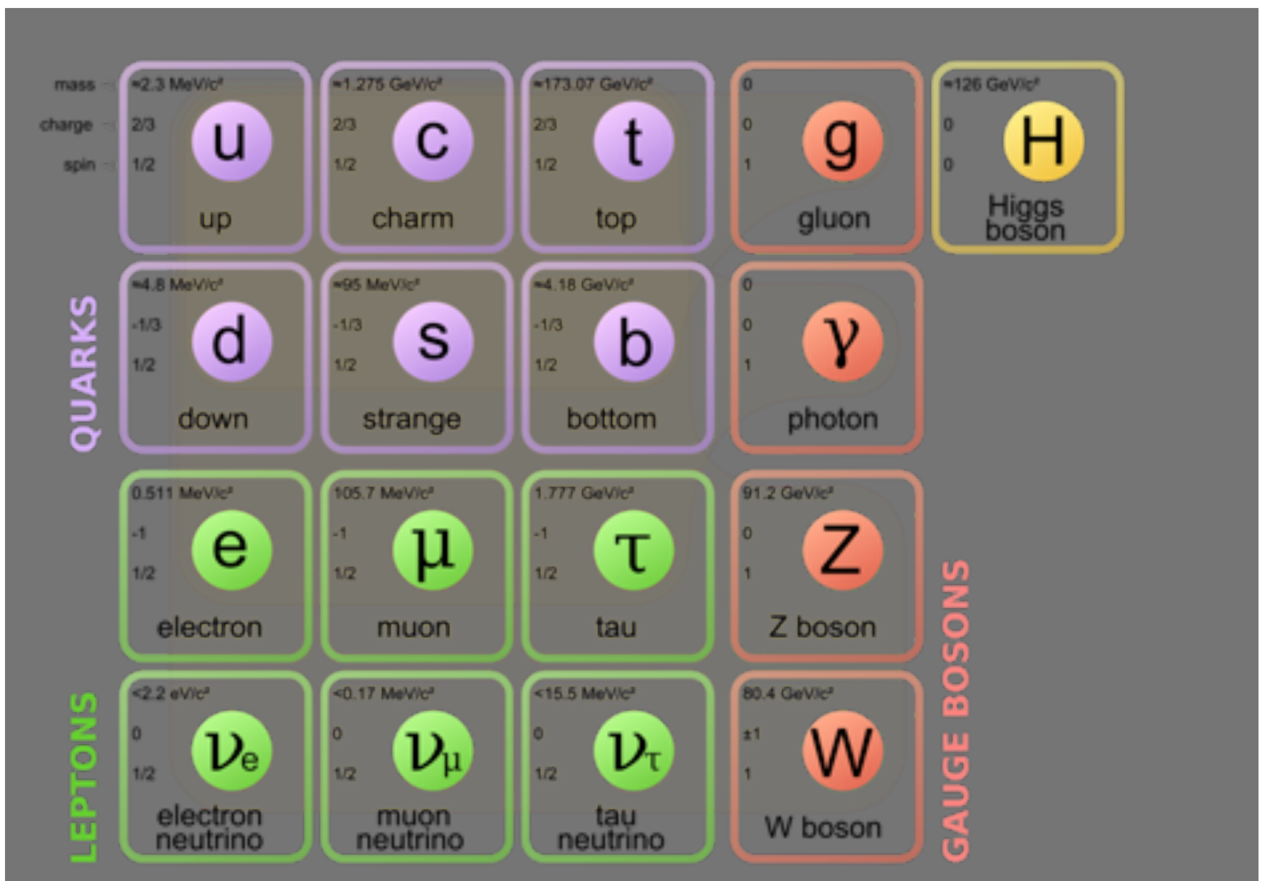
> This brings us back to our opening remarks. There has always been difficulty in explaining the high helium content of cosmic material in terms of ordinary stellar processes. The mean luminosities of galaxies come out appreciably too high on such a hypothesis. The arguments presented here make it clear, we believe, that the helium was produced in a far more dramatic way. Either the Universe has had at least one high-temperature, high-density phase, or massive objects must play (or have played) a larger part in astrophysical evolution than has hitherto been supposed.

Wagoner, Fowler, Hoyle 1966 began one of the first modern BBN computations

Beyond Nucleosynthesis

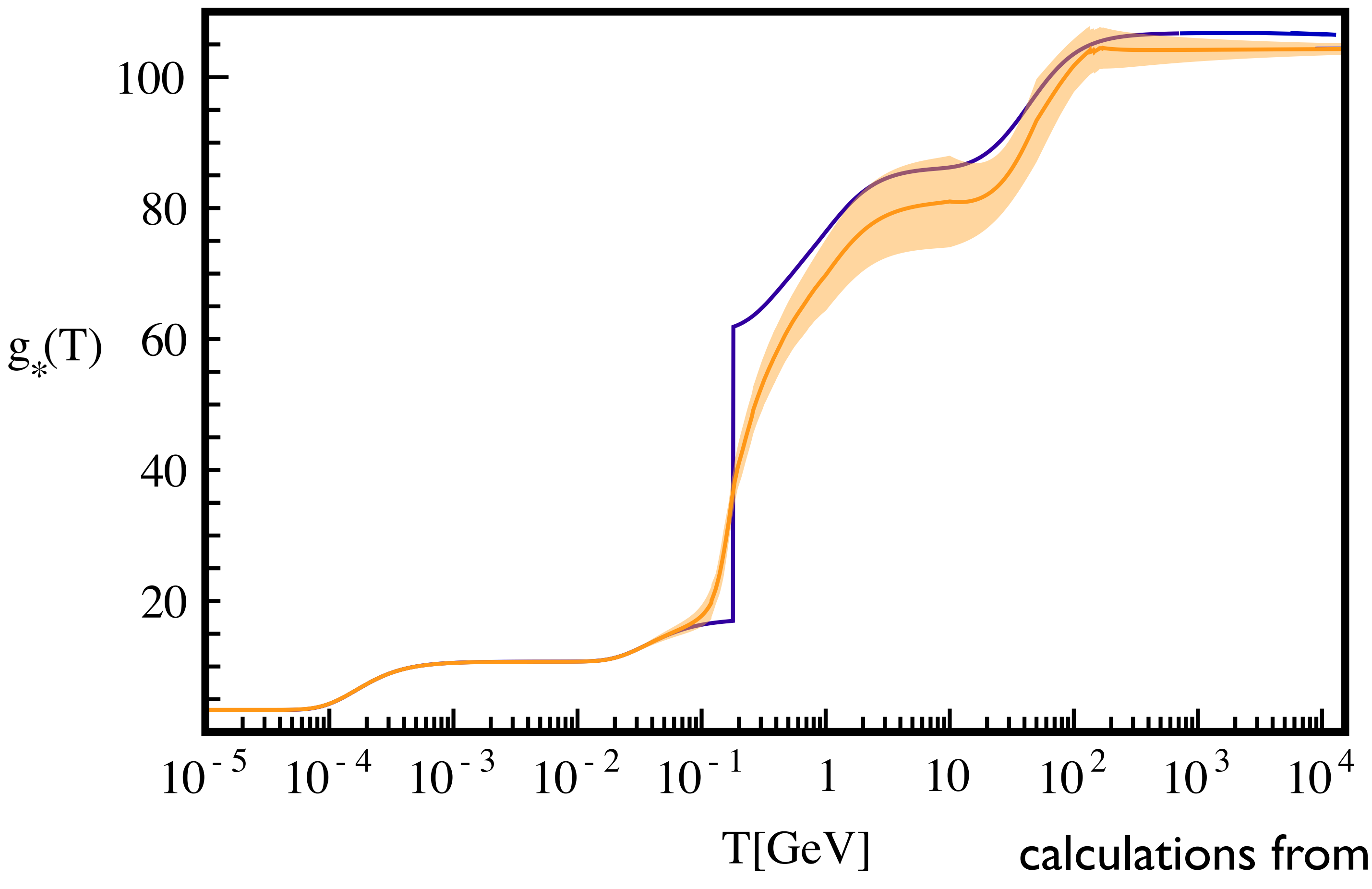
Nucleosynthesis is the earliest epoch for which we have direct evidence in the form of abundances of light elements

Statements about earlier epochs are extrapolations based on our understanding of particle physics



Beyond Nucleosynthesis

We can extrapolate the thermal history back to the electroweak phase transition



calculations from (1803.01038)

and we can speculate what lies beyond

Beyond Nucleosynthesis

This leaves us with at least two important questions

- What is the dark matter?
- What is the origin of the baryon asymmetry?

Dark Matter

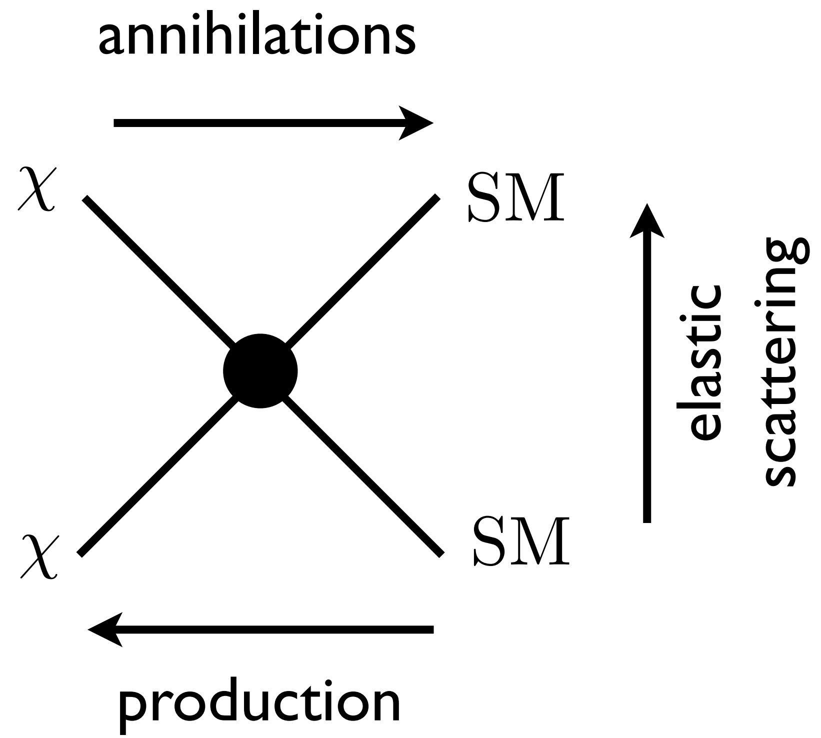
While we have good evidence for dark matter from galaxy clusters, rotation curves of spirals, CMB, we don't know what it is. Some popular ideas are

- thermal WIMPs
- axions
- dark photons
- asymmetric dark matter, self-interacting dark matter, primordial black holes, dark photons, WIMPless dark matter, ...

Dark Matter

Thermal WIMP

Weakly interacting particle that was in thermal equilibrium early on, then froze out and decoupled.



Dark Matter

Freeze out

Annihilations are described by

$$\frac{dn_\chi a^3}{dt} = -a^3 \langle \sigma_{\rm ann} v \rangle (n_\chi^2 - n_{\chi,eq}^2)$$

with

$$n_{\chi,eq} = g \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{E_p/kT} \pm 1}$$

As long as $n_\chi \langle \sigma_{\rm ann} v \rangle \gg H$, we have $n_\chi \approx n_{\chi,eq}$

Dark Matter

As $T \lesssim m_\chi$, $n_{\chi,eq}$ decays rapidly

$$\frac{dn_\chi a^3}{dt} \approx -a^3 \langle \sigma_{\rm ann} v \rangle n_\chi^2$$

The solution is

$$\frac{1}{n_\chi a^3} = \frac{1}{n_\chi(t_i) a^3(t_i)} + \int_{t_i}^{t} dt \frac{\langle \sigma_{\rm ann} v \rangle}{a^3}$$

which approaches a constant because the integral converges as $t \to \infty$.

Dark Matter

For a crude estimate of the freeze-out abundance note that

$$n_{\chi,f}\langle\sigma_{\rm ann}v\rangle \approx H_f$$

implies

$$n_{\chi,f} \propto \frac{T_f^2}{M_P\langle\sigma_{\rm ann}v\rangle}$$

Then

$$\Omega_\chi \approx \frac{m_\chi n_{\chi,f}}{\rho_0}\frac{T_{\rm CMB}^3}{T_f^3} \approx \frac{m_\chi}{T_f}\frac{1}{\langle\sigma_{\rm ann}v\rangle}\frac{T_{\rm CMB}^3}{\rho_0 M_P}$$

Dark Matter

For a crude estimate of the freeze-out abundance note that

$$n_{\chi,f}\langle\sigma_{\text{ann}}v\rangle \approx H_f$$

implies

$$n_{\chi,f} \propto \frac{T_f^2}{M_P\langle\sigma_{\text{ann}}v\rangle}$$

Then

$$\Omega_\chi \approx \frac{m_\chi n_{\chi,f}}{\rho_0}\frac{T_{\text{CMB}}^3}{T_f^3} \approx \frac{20}{\langle\sigma_{\text{ann}}v\rangle}\frac{T_{\text{CMB}}^3}{\rho_0 M_P}$$

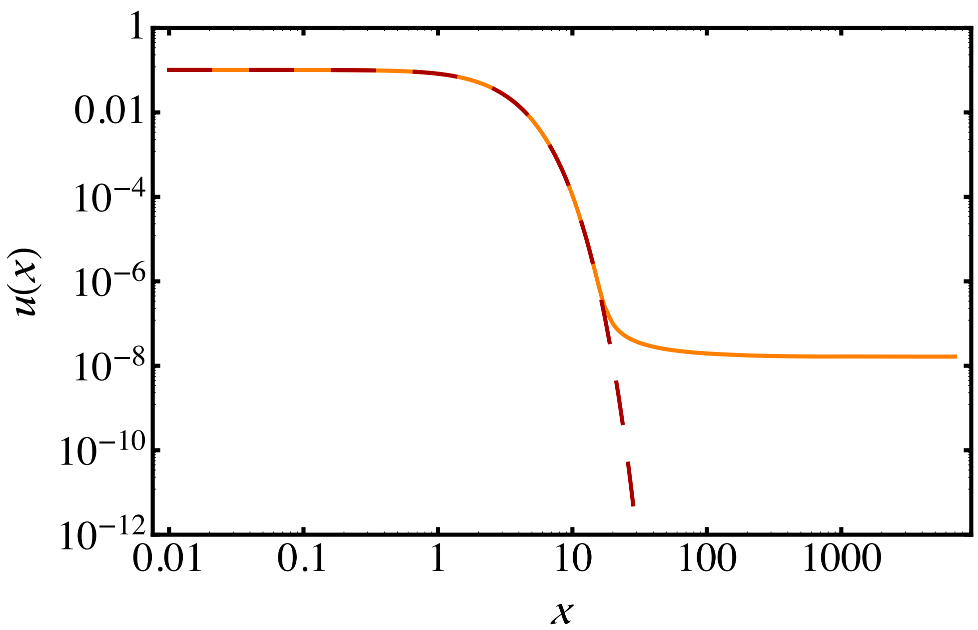
For weak scale cross-sections this is consistent with the observed abundance.

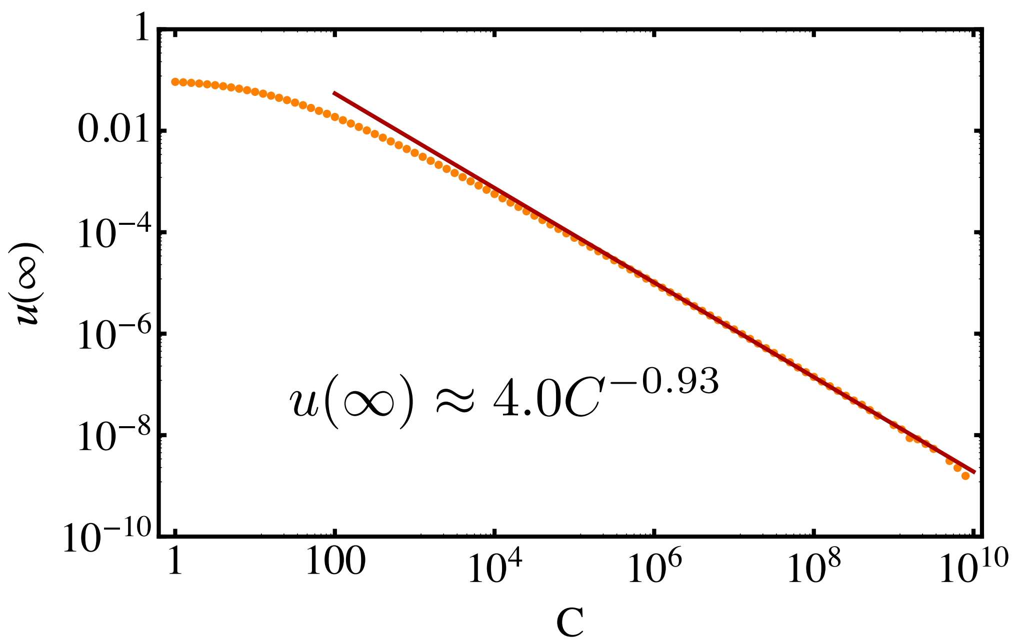
Dark Matter

We can also solve it numerically

In terms of $u = \dfrac{n_\chi}{T^3}$ and $x = \dfrac{m_\chi}{T}$

we have $$\frac{du(x)}{dx} = -\frac{C}{x^2}\left(u^2(x) - u_{\rm eq}^2(x)\right)$$





Dark Matter

Kinetic decoupling

The elastic scattering rate per dark matter particle is not affected by the drop in number density and the dark matter particles remain kinetically coupled after freeze-out.

For coupling to relativistic degrees of freedom

$$\frac{dN_\chi(\mathbf{p})}{dt} = \omega_{\mathrm{r}}(t) \frac{\partial}{\partial p_i} \left[p_i N_\chi(\mathbf{p}) + a^2 m_\chi T \frac{\partial}{\partial p_i} N_\chi(\mathbf{p}) \right]$$

Dark Matter

Kinetic decoupling

For a phase space distribution $N_\chi(\mathbf{p})$ we can define the temperature

$$T_\chi = \frac{1}{3m_\chi n_{\chi\,\mathrm{eq}}} \int \frac{d^3p}{(2\pi)^3} p^2 N_\chi(\mathbf{p})$$

This temperature obeys

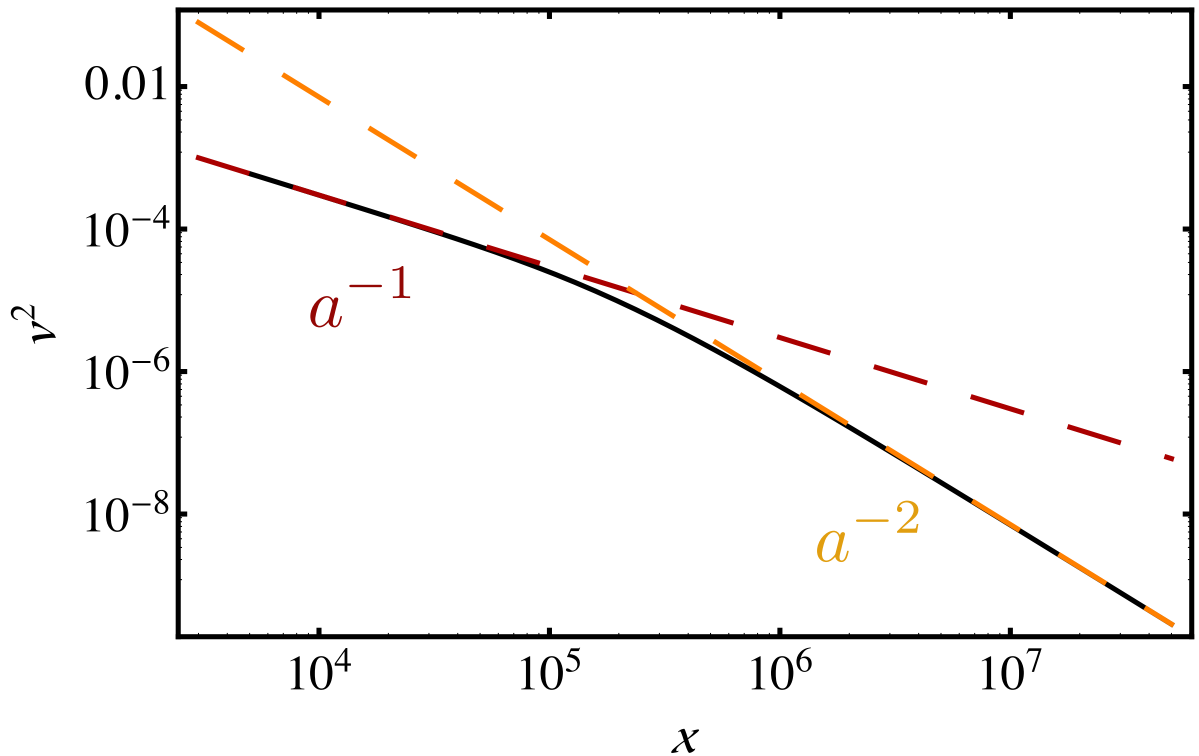
$$\frac{1}{a^2}\frac{d}{dt}\left(a^2 T_\chi\right) = 2\omega_{\mathrm{r}}(t)\left(T - T_\chi\right)$$

At early times $\omega_{\mathrm{r}}(t) \gg H$ so $T_\chi \approx T$

Dark Matter

At late times $\frac{1}{a^2}\frac{d}{dt}\left(a^2 T_\chi\right) \approx 0$ so $T_\chi \propto \frac{1}{a^2}$

In terms of the dimensionless variable $\overline{v^2} = \frac{3T_\chi}{m_\chi}$



Baryon Asymmetry

What is the cause of the matter-anti-matter asymmetry?

Perhaps the most satisfactory answer would be that the universe was initially symmetric but some process generated an asymmetry.

Any such process must satisfy Sakharov's criteria

- Baryon number violation
- C, CP violation
- departure from thermal equilibrium

Baryon Asymmetry

<u>Baryon number violation</u>

Baryon number is an accidental symmetry in the standard model

- Relevant operators respect baryon number
- There are irrelevant operators that violate baryon number
- Non-perturbative effects (instantons and sphalerons) violate baryon number

Baryon Asymmetry

<u>C, CP violation</u>

The standard model violates C, and it violates CP in the quark sector, but the CP violation is too small and additional sources of CP violation are needed.

The CP violation could arise in the neutrino sector or the Higgs sector.

<u>Departure from thermal equilibrium</u>

Departure from equilibrium can come in many forms, often it is realized (in models) through the decay of a heavy particle.

Baryon Asymmetry

Schematic example

Neutral particle X decays into final state with baryon number B with branching ratio r and final state with baryon number -B with branching ratio (1-r)

$$\Delta B = rB - (1-r)B$$

If C and CP are preserved, r=1/2, but if C and CP are violated general r are allowed.

Out of equilibrium if $\Gamma \sim H$ when $T \lesssim m_X$.

Baryon Asymmetry

With

$$\Gamma = \alpha_X m_X$$

and

$$H = \sqrt{\frac{8\pi G}{3}\frac{\pi^2}{30}g_*(kT)^4}$$

we have

$$\alpha_X m_X = \sqrt{\frac{8\pi G}{3}\frac{\pi^2}{30}g_*(kT_X)^4} \lesssim \sqrt{\frac{8\pi G}{3}\frac{\pi^2}{30}g_* m_X^2}$$

or

$$m_X \gtrsim \alpha_X M_{\mathrm{P}} g_*^{-1/2}$$

Typical mass scale for grand unified theories

Baryon Asymmetry

This is just a simple schematic example and several other ideas exist

- leptogenesis
- Affleck-Dine
- electroweak baryogenesis
- ...

Beyond the hot big bang

A hot big bang is very successful at describing the universe around us, but some questions take us beyond it

- Why is the CMB so isotropic?
- What generated the primordial perturbations? see in

Additional related questions

- Why is the universe so flat?
- Why do we not see monopoles?

Beyond the hot big bang

Horizon problem

For a medium to reach thermal equilibrium different regions must be in causal contact.

In a big bang, the age of the universe is finite and signals traveled a finite distance.

$$d_h = a_L r_h = a_L \int_0^{t_L} \frac{dt}{a(t)}$$

The angular size in the CMB is

$$\theta_h = \frac{d_h}{d_A}$$

Beyond the hot big bang

With

$$d_A = a_L \int_{t_L}^{t_0} \frac{dt}{a(t)}$$

For the observed values of cosmological parameters

$$\theta_h \approx 0.02 \approx 1^\circ$$

And we expect fluctuations of order unity on degree scales, inconsistent with observations.

Beyond the hot big bang

Diagrammatically